PMM U.S.S.R.,Vol.50,No.1,pp.106-111,1986
Printed in Great Britain

0021-8928/86 $10.00+0.00

CLASSIFICATION OF FREE PIEZOCERAMIC SHELL VIBRATIONS*

N.N. ROGACHEVA

Free vibrations of piezoceramic shells of arbitrary shape which have first been polarized along one of the families of coordinates lines of the middle surface are examined. The various kinds of vibrations are classified by an asymptotic method, and approximate equations and boundary conditions corresponding to each kind of vibrations are obtained.

1. We select coordinate lines α_1 and α_2 coincident with the lines of curvature on the shell middle surface. We consider the piezoceramic shell to be polarized first along the α_2 lines, and the shell front surfaces to have no electrodes.

Let us write down the initial system of equations.

Equilibrium equations

$$\frac{1}{A_i}\frac{\partial T_i}{\partial \alpha_i}+\frac{1}{A_j}\frac{\partial S}{\partial \alpha_j}+qk_j(T_i-T_j)-q2k_iS-qp\frac{N_i}{R_i}+q2h\rho\Omega^2u_i=0 \tag{1.1}$$

$$\frac{T_1}{R_1}+\frac{T_2}{R_2}+p\left(\frac{1}{A_1}\frac{\partial N_1}{\partial \alpha_1}+\frac{1}{A_2}\frac{\partial N_2}{\partial \alpha_2}+qk_2N_1+qk_1N_2\right)+2h\rho\Omega^2w=0 \tag{1.2}$$

$$N_i=\frac{1}{A_i}\frac{\partial G_i}{\partial \alpha_i}-\frac{1}{A_j}\frac{\partial H}{\partial \alpha_j}+qk_j(G_i-G_j)-2k_iH \tag{1.3}$$

Piezoelasticity relationships and electrostatics equations

$$T_i=2hn_{ii}(\varepsilon_i+\nu_i\varepsilon_j)-2hc_iE_2,\quad S=\frac{2h}{s_{44}{}^E}(\omega-d_{15}E_1) \tag{1.4}$$

$$G_i=-\frac{2h^3n_{ii}}{3}(\varkappa_i+\nu_i\varkappa_j),\quad H=\frac{2h^3}{3s_{44}{}^E}\tau \tag{1.5}$$

$$D_1=\varepsilon_{11}{}^TE_1+\frac{d_{15}}{2h}S,\quad D_2=\varepsilon_{33}{}^TE_2+\frac{d_{31}}{2h}T_1+\frac{d_{33}}{2h}T_2 \tag{1.6}$$

$$\varepsilon_{11}{}^T\frac{\partial}{\partial \alpha_1}\frac{A_2}{A_1}\frac{\partial \psi}{\partial \alpha_1}+\varepsilon_{33}{}^T\frac{\partial}{\partial \alpha_2}\frac{A_1}{A_2}\frac{\partial \psi}{\partial \alpha_2}=\frac{d_{15}}{2h}\frac{\partial}{\partial \alpha_1}A_1S+\frac{d_{31}}{2h}\frac{\partial}{\partial \alpha_2}A_1T_1+\frac{d_{33}}{2h}\frac{\partial}{\partial \alpha_2}A_1T_2 \tag{1.7}$$

$$E_i=-\frac{1}{A_i}\frac{\partial \psi}{\partial \alpha_i} \tag{1.8}$$

Strain-displacement formulas

$$\varepsilon_i=\frac{1}{A_i}\frac{\partial u_i}{\partial \alpha_i}+qk_iu_j-\frac{w}{R_i} \tag{1.9}$$

$$\omega=\frac{1}{A_1}\frac{\partial u_2}{\partial \alpha_1}+\frac{1}{A_2}\frac{\partial u_1}{\partial \alpha_2}-q(k_1u_1+k_2u_2)$$

$$\varkappa_i=-\frac{1}{A_i}\frac{\partial \gamma_i}{\partial \alpha_i}-qk_i\gamma_j,\quad \gamma_i=-\frac{1}{A_i}\frac{\partial w}{\partial \alpha_i}-q\frac{u_i}{R_i} \tag{1.10}$$

$$\tau=-\frac{1}{A_i}\frac{\partial \gamma_i}{\partial \alpha_i}+qk_i\gamma_i+q\frac{1}{R_i}\left(\frac{1}{A_j}\frac{\partial u_i}{\partial \alpha_j}-k_ju_j\right)$$

The following notation was introduced here

$$n_{11}=\frac{s_{33}{}^E}{\delta},\quad n_{12}=n_{21}=\frac{s_{13}{}^E}{\delta},\quad n_{22}=\frac{s_{11}{}^E}{\delta},$$

$$c_1=\frac{d_{31}s_{33}{}^E-d_{33}s_{13}{}^E}{\delta}$$

$$c_2=\frac{d_{33}s_{11}{}^E-d_{31}s_{13}{}^E}{\delta},\quad \delta=s_{11}{}^Es_{33}{}^E-(s_{13}{}^E)^2,$$

*Prikl.Matem.Mekhan.,50,1,147-154,1986

$$\nu_1 = \frac{n_{12}}{n_{11}}, \quad \nu_2 = \frac{n_{12}}{n_{22}}, \quad k_i = \frac{1}{A_i A_j} \frac{\partial A_i}{\partial \alpha_j}$$

Here and henceforth, each equation containing the subscripts i and j should be considered as two equations, the first being obtained if $i=1$, $j=2$ and the second $i=2$, $j=1$. The numbers p, q that will be used below should be considered to be equal to one.

It is taken into account in (1.1) and (1.2) that the shells performs vibrations according to the law $e^{-i\Omega\tau}$ where $i=\sqrt{-1}$, τ is time, and Ω the angular frequency of the vibrations.

The equilibrium equations and the strain-displacement relationships are the same as in the theory of non-electrical shells /1/. The two-dimensional piezoelasticity relationships and electrostatics equations are obtained in /2/.

The notation used agrees with that employed in /1, 2/.

As is shown in /2, 3/, in general the two-dimensional problem for shells with preliminary polarization and frontal surfaces without electrodes does not split into mechanical and electrical problems. The system of differential equations of the theory of electroelastic shells is of ninth order; consequently, five boundary conditions should be satisfied on each edge of the shell, four mechanical that agree with the conditions taken in the theory of non-electrical shells, and one electrical condition.

We will limit ourselves to considering shells with two kinds of electrical conditions at the edges. On the edge without electrodes $\alpha_i = \alpha_{i0}$ the electrical induction vector component normal to the endface surface should be zero in a vacuum or in air $D_i = 0$. On the edge $\alpha_i = \alpha_{i0}$ with short-circuited electrodes, the electrical potential equals zero: $\psi = 0$.

2. The free steady vibration spectrum will be investigated by an asymptotic method, just as was done in /4/. Asymptotic integration of the piezoelastic shell equations reduces, as a rule, to two iteration processes, the fundamental that results in a principal boundary value problem (PP), which consists of integrating the degenerate problem while satisfying certain boundary conditions (the natural frequencies are determined, in particular, from the equations of the principal problem), and an additional boundary value problem (AP corresponding to an iteration process that allows reduction of the rasidual in the remaining boundary conditions.

The free vibrations of an arbitrary piezoceramic shell can be subdivided into quasitransverse, quasitangential, and ultra-low frequency Rayleigh vibrations. The terminology of the theory of non-electrical shells /4/ is retained here despite the fact that the PP and AP equations include electrical quantities.

Let us make the replacement of the independent variables α_i, which is usual for asymptotic methods, by means of the formulas

$$\alpha_i = \eta^t R \xi_i \tag{2.1}$$

Here η is the shell relative half-thickness, R is the characteristic dimension, and t is the index of variability of the electroelastic state. The dimensionless coordinates ξ_i are selected in such a manner that differentiation with respect to them does not result in a substantial increase or decrease in the desired functions.

We introduce dimensionless quantities of one order (denoted by asterisks) in place of the desired quantities as follows:

$$\frac{u_i}{R} = \eta^{t+r} u_{i*}, \quad \frac{w}{R} = \eta^0 w_* \tag{2.2}$$
$$\left(\frac{T_i}{2hn_{11}}, \frac{S}{2hn_{11}}\right) = \eta^{c+r}(T_{i*}, S_*),$$
$$\left(\frac{G_i}{2hRn_{11}}, \frac{H}{2hRn_{11}}\right) = \eta^{2-2t} \quad (G_{i*}, H_*)$$
$$\frac{N_i}{2hn_{11}} = \eta^{2-3t} N_{i*}, \quad \frac{\varepsilon_{11}{}^T}{d_{15} n_{11} R}\psi = \eta^{t+r+c}\psi_*, \quad \frac{\rho\Omega^2 R^2}{n_{11}} = \eta^{b+t}\Omega_*{}^2$$

The numbers r, b, c take different values depending on the kind of vibrations.

For each kind of vibration the asymptotic representation of the desired quantities is selected in such a manner that it will have physical meaning and result, to a first approximation, in an incontrovertible system of equations in which the number of unknowns equals the number of equations. Moreover, the PP and AP boundary conditions should be separated in such a manner that the boundary conditions for the additional problem will be inhomogeneous while the residuals that will again appear in the boundary conditions of the partial problem after solution of the additional problem will be small.

We substitute (2.1) and (2.2) into (1.1)-(1.10). We consequently obtain equations in which the order of each term in the equation is determined explicitly by the factor η in front, to a certain power.

3. We will mean by quasiperiodic vibrations with small variability those for which the index of variability of the electroelastic state is less than $^1/_2$ and the deflection w is substantially greater than the displacements u_1, u_2. The numbers r, c, b should be selected as follows:

$$r = b = c = 0 \tag{3.1}$$

As a reult of substituting the asymptotic expressions (2.2) and (3.1) and replacing the variables (2.1), we obtain the system of PP equations from (1.1)–(1.10) after neglecting small terms. It includes the membrane equilibrium Eqs.(1.1) and (1.2) (in which it is necessary to set $p = 0$, $q = 1$), the piezoelasticity relationships and electrostatics Eqs.(1.4), (1.6)–(1.8), and (1.9). This system of equations is to determine the displacements, tangential forces and natural frequencies. It is the initial approximation for the fundamental iteration process.

The PP describes the membrane electroelastic state. Near the edges it should be supplemented by the electroelastic state with variability $^1/_2$ in a direction orthogonal to the edge, and t (t is the index of variability in the PP) along the edge. This state of stress is determined by using an additional iteration process. We will write down its asymptotic expression and the first approximation equations.

Near the edge $\alpha_i = \alpha_{i0}$ the dimensionless quantities with the asterisk and the dimensionless coordinates

$$\begin{gathered}
\frac{u_i}{R} = \eta^{1/2} u_{i*}, \quad \frac{u_j}{R} = \eta^{1-t} u_{j*}, \quad \frac{w}{R} = w_* \qquad (3.2) \\
\left(\frac{G_1}{2hRn_{11}}, \frac{G_2}{2hRn_{11}}\right) = \eta^1 (G_{1*}, G_{2*}), \quad \frac{H}{2hRn_{11}} = \eta^{3/2-t} H_* \\
\frac{N_i}{2hn_{11}} = \eta^{1/2} N_{i*}, \quad \frac{N_j}{2hn_{11}} = \eta^{1-t} N_{j*}, \quad \frac{T_j}{2hn_{11}} = \eta^0 T_{j*} \\
\frac{S}{2hn_{11}} = \eta^{1/2-t} S_*, \quad \frac{T_i}{2hn_{11}} = \eta^{1/2} T_{i*}, \quad d_i \psi = \eta^{1-t} \psi_* \\
\alpha_i = \eta^{1/2} R \xi_i, \quad \alpha_j = \eta^t R \xi_j \qquad (3.3)
\end{gathered}$$

should be introduced in place of the desired quantities.

Taking account of (3.2) and (3.3), we obtain the following fundamental relationships of the additional electroelastic state to a first approximation:

$$\begin{gathered}
\frac{\partial^4 w}{\partial \alpha_i^4} + 4g_i^4 w = 0 \qquad (3.4) \\
u_i = -\left(\frac{1}{R_i} + \frac{\nu_i + b_i}{R_j}\right) \frac{A_i}{4g_i^4} \frac{\partial^3 w}{\partial \alpha_i^3}, \quad \varkappa_i = \frac{1}{A_i^2} \frac{\partial^2 w}{\partial \alpha_i^2} \\
T_j = -2hn_{jj} \frac{a_i}{R_j} w, \quad \frac{\partial S}{\partial \alpha_i} = -\frac{1}{A_j} \frac{\partial}{\partial \alpha_j} (A_i T_j) \\
G_i = -\frac{2h^3 n_{ii}}{3} \varkappa_i, \quad G_j = \nu_i G_i, \quad N_i = \frac{1}{A_i} \frac{\partial G_i}{\partial \alpha_i}, \quad \frac{1}{A_i} \frac{\partial \psi}{\partial \alpha_i} = F_i
\end{gathered}$$

The following notation is used in (3.2) and (3.4).

$$\begin{gathered}
4g_i^4 = \frac{3A_i^4}{h^2 n_{11}} \left(\frac{n_{jj} a_i}{R_j^2} - \rho \Omega^2\right) \qquad (3.5) \\
a_1 = 1 - \nu_1 \nu_2, \quad a_2 = \frac{a_1 \varepsilon_{33}^T}{\varepsilon_{33}^T + d_{31} (\nu_2 c_2 - c_1)}, \quad b_1 = 0, \quad b_2 = \frac{c_2 d_{31} n_{11} a_2}{n_{22} \varepsilon_{33}^T} \\
d_1 = \frac{\varepsilon_{11}^T}{d_{15} n_{11} R}, \quad d_2 = \frac{\varepsilon_{33}^T}{d_{31} n_{11} R}, \quad F_1 = \frac{d_{15} - d_{33}}{2h\varepsilon_{11}^T} S, \quad F_2 = \frac{d_{31}}{2h\varepsilon_{33}^T} T_1
\end{gathered}$$

At the edge $\alpha_1 = \alpha_{10}$ we should set $i = 1$, $j = 2$ in the additional electroelastic state formulas (3.2)–(3.5), and $i = 2, j = 1$ at the edge $\alpha_2 = \alpha_{20}$. The frequency Ω in (3.5) is a known quantity. It is found when solving the PP.

The nature of the solution of (3.4) depends on the sign of g_i^4: if $g_i^4 > 0$ then the solution damps out with distance from the edge, if $g_i^4 < 0$, the solution is oscillating. The case $g_i^4 = 0$ requires special examination.

The additional electroelastic state is described by equations analogous to the equations of the additional state of stress of non-electric shells. The governing equation and formulas for the mechanical quantities agree with known formulas of non-electric shell theory apart from constant coefficients. In the statics case (3.4) reduces to the equations of the simple edge effect for piezoceramic shells.

The PP equations enable us to satisfy three conditions on each edge. The residuals that occur in the two discarded boundary conditions can be eliminated by using the arbitrariness of the additional electroelastic state.

The boundary conditions for the PP and AP are obtained by the scheme presented in /5/. By separating the complete boundary conditions we obtain two tangential mechanical conditions for the PP that agree with the conditions for the membrane theory of non-electric shells, and one electrical condition. The solution of the AP reduces the residual in the two remaining mechanical non-tangential conditions. For instance, on the rigidly framed non-electrical edge

$\alpha_1 = \alpha_{10}$ the complete boundary conditions are separated as follows:

$$u_1^{(b)}=0,\quad u_2^{(b)}=0,\quad D_1^{(b)}=0,\quad w^{(c)}=-w^{(b)},\quad \gamma_1^{(c)}=0 \tag{3.6}$$

The superscripts b and c denote whether the quantities belong to the PP and AP, respectively. The first three conditions in (3.6) are satisfied in the PP solution and the last two boundary conditions in the AP solution.

By analysing all the errors made in deriving the equations and the boundary conditions, it can be shown that the PP and AP are constructed with an error of the order of $\eta^{1/2-t}$.

4. We consider quasitransverse vibrations with high variability $t > 1/2$. The asymptotic expression of the principal and additional problems is obtained from (2.2) for $r = 0,\ b = 2 - 4t,\ c = 0$. Taking account of the asymptotic expression used, we obtain the PP and AP equations from system (1.1)–(10) with an accuracy of $0\,(\eta^t + \eta^{2-2t})$.

The system of PP equations

$$\frac{1}{A_1}\frac{\partial N_1}{\partial\alpha_1}+\frac{1}{A_2}\frac{\partial N_2}{\partial\alpha_2}+2h\rho\Omega^2 w=0,\quad N_i=\frac{1}{A_i}\frac{\partial G_i}{\partial\alpha_i}-\frac{1}{A_j}\frac{\partial H}{\partial\alpha_j} \tag{4.1}$$

$$H=\frac{2h^3}{3s_{44}^E}\tau,\quad G_i=-\frac{2h^3 n_{ii}}{3}(\varkappa_i+\nu_i\varkappa_j),\quad \varkappa_i=-\frac{1}{A_i}\frac{\partial\gamma_i}{\partial\alpha_i},$$

$$\tau=-\frac{1}{A_j}\frac{\partial\gamma_i}{\partial\alpha_j},\quad \gamma_i=-\frac{1}{A_i}\frac{\partial w}{\partial\alpha_i}$$

differs from the corresponding system of equations of the theory of non-electric shells only in the meaning of the constant coefficients $n_{ii},\ \nu_i,\ s_{44}^E$ in the elasticity relationships for the moments. They describe the bending vibrations.

The equations of the additional problem are written as follows:

$$\frac{1}{A_i}\frac{\partial T_i}{\partial\alpha_i}+\frac{1}{A_j}\frac{\partial S}{\partial\alpha_j}=0,\quad \varepsilon_i=\frac{1}{A_i}\frac{\partial u_i}{\partial\alpha_i}-\frac{w}{R_i} \tag{4.2}$$

$$\omega=\frac{1}{A_1}\frac{\partial u_2}{\partial\alpha_1}+\frac{1}{A_2}\frac{\partial u_1}{\partial\alpha_2}$$

$$\frac{\varepsilon_{11}^T}{A_1^2}\frac{\partial^2\psi}{\partial\alpha_1^2}+\frac{\varepsilon_{33}^T}{A_2^2}\frac{\partial^2\psi}{\partial\alpha_2^2}=\frac{d_{15}}{2h}\frac{1}{A_1}\frac{\partial S}{\partial\alpha_1}+\frac{1}{2h}\frac{1}{A_2}\frac{\partial}{\partial\alpha_2}(d_{31}T_1+d_{33}T_2)$$

To obtain a closed system, (1.4), (1.6) and (1.8) must be added to these equations.

After the principal system of Eqs.(4.1) has been integrated, the solution of the system of additional problem equations has been found, it is here considered that w is a known function found in solving the principal problem.

The order of the system of PP equations is four, and that of the AP equations is six. The asymptotic separation of the boundary conditions executed by the scheme in /5/ shows that two non-tangential mechanical boundary conditions should be satisfied in integrating the PP equations on each edge, and two tangential mechanical boundary conditions and one electrical condition for the AP. For instance, on a rigidly framed edge $\alpha_1=\alpha_{10}$ without electrodes, the boundary conditions are separated in the following manner: $w^{(b)} = 0,\ \gamma_1^{(b)} = 0,\ u_1^{(c)} = 0,\ u_2^{(c)} = 0,\ D_1^{(c)} = 0$. As a result of solving the PP and the AP, the desired quantities will be determined with an error $O\,(\eta^t)$, where $t > 1/2$.

The quasitransverse vibrations with variability $t = 1/2$ are described to a first approximation by equations which are a generalization of the dynamical equations of the theory of non-electric shells with high variability. These equations are obtained if we set $p = 0,\ q = 1$ in (1.1)–(1.10). Moreover, to a first approximation the coefficients of the first and second quadratic forms of the middle surface should be considered constants in $\alpha_1,\ \alpha_2$.

In the case of quasitransverse vibrations with variability $t = 1/2$ the problem does not separate into PP and AP even in the initial approximation; hence, all five boundary conditions should be taken into account on each edge when integrating the simplified system of equations obtained. The error of the first approximation is a quantity $O\,(\eta^{1/2})$.

5. The quasitangential vibrations are characterized by the fact that their tangential displacements are substantially greater than the deflections. The asymptotic expression (2.2) holds for the desired quantities for $r = -2t,\ b = -2t,\ c = 0$. It is suitable for $0 < t < 1$. Asymptotic integration of (1.1)–(1.10) results in PP and AP. The system of PP equations consists of the equilibrium Eqs.(1.1) in which the transverse forces should be discarded, the piezoelasticity relationships (1.4), formulas (1.6)–(1.8), and the tangential strain-displacement relationships in which w should be discarded. This system of equations can be considered as the equations of the plane theory of piezoelasticity. The system of equations is of sixth order, consequently, three boundary conditions should be posed on each edge.

After the PP has been solved, all the remaining desired quantities can be predetermined from the remaining unused Eqs.(1.2), (1.10), (1.5), (1.3) by using direct action. From Eq. (1.2), simplified by taking account of the asymptotic expression, we must find

$$w = -\frac{1}{2h\rho\Omega^2}\left(\frac{T_1}{R_1} + \frac{T_2}{R_2}\right)$$

and then the bending and twisting strains, the moments, and the transverse forces should be determined successively from (1.10), (1.5), (1.3). The PP quasitransverse vibrations Eqs.(3.4) remain valid even for the PP quasitangential vibrations. The exception is the formula for $4g_i^4$ which must be taken in the following form:

$$4g_i^4 = -\frac{3A_i^4}{h^2 n_{ii}}\rho\Omega^2$$

The variability of the electroelastic state described by the PP in an orthogonal direction to the edge is $(1+t)/2$ (t is the variability of the electroelastic state determined by the AP). The boundary conditions are separated in exactly the same way as in the case of quasitransverse vibrations with low variability. The PP solution satisfies two tangential mechanical conditions and one electrical condition; the residues being formed here are reduced in the non-tangential mechanical conditions because of the arbitrariness of the PP.

6. If the shell edges are free, then the shell can perform ultralow Rayleigh-type vibrations. An asymptotic analysis of the equations shows that the complete problem dissociates into the PP and AP. The asymptotic expression (2.2) for $r=0$, $c=b=2-4t$ holds for the PP quantities. The system of PP equations consists of the Eqs.(1.1)-(1.3) ($p=q=1$) (1.5)-(1.8), and the equations which replace relations (1.4).

$$\varepsilon_i = 0, \quad \omega = 0 \tag{6.1}$$

Note that the electrical quantities are only in (1.6)-(1.8). The remaining PP equations form a complete set of equations with respect to the mechanical quantities that agrees with the system of equations of free non-electric shells apart from a constant coefficient. Hence, the PP equations should be integrated in two stages: it is first necessary to find the solution of the mechanical problem, then to integrate (1.7) with respect to the electrical potential ψ by considering the forces as known. The quantities E_i, D_i are defined by means of ψ and the forces by direct actions.

The AP equations are the equations of a simple edge effect. They are obtained from (3.4) for $\omega = 0$.

We separate the boundary conditions into boundary conditions for the PP and a simple edge effect. All the desired quantities in the boundary conditions should be represented in the form of a sum of PP and simple edge effect quantities, taking their asymptotic representation into account. For instance, the boundary conditions on the edge $\alpha_1 = \alpha_{10}$ without electrodes are written in the form

$$\eta^{2-4t}T_{1*}^{(b)} + \eta^{a+1/2}T_{1*}^{(c)} = 0, \quad \eta^{2-4t}S_*^{(b)} + \eta^{a+1/2-t}S_*^{(c)} = 0$$
$$\eta^{2-4t}D_{1*}^{(b)} + \eta^{a+1/2-t}D_{1*}^{(c)} = 0, \quad \eta^{2-2t}G_{1*}^{(b)} + \eta^{a+1}G_{1*}^{(c)} = 0$$
$$\eta^{2-3t}N_{1*}^{\prime(b)} + \eta^{a+1/2}N_{1*}^{(c)} = 0$$

Here N_1' is the reduced edge force /1/.

The values of the simple edge effect are determined from the homogeneous equations, consequently, there is the scale factor η^a before them, where a is selected in such a manner that inhomogeneous boundary conditions hold for a simple edge effect. In this case a should be taken equal to $(1-2t)$. Then the boundary conditions for the simple edge effect take the form

$$G_1^{(c)} = -G_1^{(b)}, \quad N_1^{(c)} = -\eta^{1/2-t}N_1^{\prime(b)}$$

As was done in /1, 6/, we express $T_1^{(c)}$, $S^{(c)}$, $D_1^{(c)}$ by using the solution of the simple edge effect in terms of $G_1^{(b)}$ and $N_1^{\prime(b)}$. Hence, we obtain two mechanical boundary conditions (6.2) for the PP on the edge $\alpha_i = \alpha_{i0}$ that agree with the corresponding conditions for the theory of non-electric shells, and one electrical condition ((6.3) on the edge with electrodes and (6.4) on the edge without)

$$T_i^{(b)} - \frac{1}{A_j}\frac{\partial}{\partial\alpha_j}\frac{1}{A_j}\frac{\partial}{\partial\alpha_j}(R_jG_i^{(b)}) + k_jR_jN_1^{\prime(b)} = 0 \tag{6.2}$$

$$S^{(b)} + k_j\frac{1}{A_j}\frac{\partial}{\partial\alpha_j}(R_jG_i^{(b)}) - \frac{1}{A_j}\frac{\partial}{\partial\alpha_j}(R_jN_i^{\prime(b)}) = 0$$

$$\psi^{(b)} = \frac{d_{15}-d_{33}}{2h\varepsilon_{11}{}^T}\frac{A_1}{A_2}\frac{\partial}{\partial\alpha_2}\left(\frac{R_2}{A_1}G_1^{(b)}\right) \quad (\alpha_1 = \alpha_{10}) \tag{6.3}$$

$$\psi^{(b)} = -\frac{R_1}{2h\varepsilon_{33}{}^T}(d_{31}N_1^{\prime(b)} + d_{33}k_1G_2^{(b)}) \quad (\alpha_2 = \alpha_{20})$$

$$D_1^{(b)} = \frac{1}{2h}\frac{1}{A_2}\frac{\partial}{\partial\alpha_2}(d_{31}k_2R_2G_1^{(b)} + d_{33}R_2N_1^{\prime(b)}) \quad (\alpha_1 = \alpha_{10}) \tag{6.4}$$

$$D_2^{(b)} = -\frac{1}{2h}\left(\frac{\varepsilon_{11}{}^Td_{31}}{\varepsilon_{33}{}^T} + d_{15}\right)\frac{1}{A_1}\frac{\partial}{\partial\alpha_1}\frac{A_2}{A_1}\frac{\partial}{\partial\alpha_1}\frac{R_1}{A_2}G_2^{(b)} \quad (\alpha_2 = \alpha_{20})$$

7. The asymptotic analysis has shown that the free vibrations of piezoceramic shells with preliminary polarization along the α_2-lines with frontal surfaces without electrodes can be subdivided into 1) quasitransverse with low variability $(0 \leqslant t < 1/_2)$, 2) quasitransverse with high variability $(1/_2 < t < 1)$, 3) quasitransverse with variability $t = 1/_2$, 4) quasitangential $(0 < t < 1)$, and 5) ultralow frequency of Rayleigh type $(0 \leqslant t < 1/_2)$.

Each of these types of vibrations is described by a corresponding set of equations. Such a classification is physically conceivable and considerably simplifies the calculation of the natural frequencies and the other desired quantities.

We note that for the same classification of the free vibrations as in the theory of non-electric shells, the systems of principal and additional boundary value problem equations differ qualitatively from the corresponding non-electric shell problems in the high order of the systems of equations, the large number of initial quantities, and the boundary conditions. Hence, the classification obtained for the free vibrations of piezoceramic shells should be considered as a generalization of the classification for the free vibrations of non-electric shells.

REFERENCES

1. GOL'DENVEIZER A.L., Theory of Elastic Thin Shells, Nauka, Moscow, 1976.
2. ROGACHEVA N.N., Equations of state of piezoceramic shells, PMM, 45, 5, 1981.
3. ROGACHEVA N.N., On boundary conditions in the theory of piezoceramic shells polarized along coordinate lines. PMM, 47, 2, 1983.
4. GOL'DENVEIZER A.L., Classification of integrals of the dynamic equations of linear two-dimensional shell theory, PMM, 37, 4, 1973.
5. ANTROPOVA N.N. and GOL'DENVEIZER A.L., Errors in constructing the fundamental state of stress and the simple edge effect in shell theory, Izv. Akad. Nauk SSSR, Mekhan., Tverd. Tela, 5, 1971.
6. CHERNYKH K.F., Linear Shell Theory, Pt. II, Izdat. Leningradsk, Gosudarst. Univ., Leningrad, 1962.

Translated by M.D.F.

PMM U.S.S.R., Vol.50, No.1, pp.111-115, 1986
Printed in Great Britain
0021-8928/86 $10.00+0.00

DIFFERENTIAL GAMES WITH VARIABLE STRUCTURE, WITH A GROUP OF PURSUERS CHASING A SINGLE TARGET*

K.V. DEMIDOV

Differential games with variables structure /1/ is which m pursuers chase a single target are considered. Sufficient conditions are given for the pursuit problems in such games to be solvable. Strategies leading to capture are devised. An example of a game for which the sufficient conditions proposed are essential, is given.

Let the motion of the i-th object $(i = 1, \ldots, m)$ prior to switchover to be described by the following equation:

$$z_i^{\cdot(1)} = C_i^{(1)}(t)\, z_i^{(1)} + f_i^{(1)}(u_i^{(1)}) - g_i^{(1)}(v), \quad t \in (0, \tau_i) \tag{1}$$
$$z_i^{(1)}(0) = z_i^{\circ}$$

and after the switchover by

$$z_i^{\cdot(2)} = C_i^{(2)}(t)\, z_i^{(2)} + f_i^{(2)}(u_i^{(2)}) - g_i^{(2)}(v), \quad t \in (\tau_i, +\infty) \tag{2}$$
$$z_i^{(2)}(\tau_i) = B_i(\tau_i)\, z_i^{(1)}(\tau_i)$$

Here $z_i^{(1)} \in R^{n_i}$, $z_i^{(2)} \in R^{m_i}$, $C_i^{(1)}(t)$ and $C_i^{(2)}(t)$ are continuous $n_i \times n_i$ - and $m_i \times m_i$ matrices respectively, the matrix $B_i(t)$ is also continuous and of dimension $m_i \times n_i$, $u_i^{(j)} \in P_i^{(j)} \subset R^{r_i^{(j)}}$, $v \in Q \subset R^q$, Q, $P_i^{(j)}$ $(j = 1, 2)$ are non-empty convex compacta. The functions $f_i^{(j)}, g_i^{(j)}$ $(i = 1, \ldots, m;\ j = 1, 2)$ depend continuously on their arguments. We specify, in the Euclidean space R^{m_i}, the terminal sets $M_i = M_i^1 + M_i^2$, where M_i^1 is a linear subspace of R^{m_i}, M_i^2 is a convex compactum from the orthogonal complement L_i^1 to the subspace M_i^1.

*Prikl.Mekhan.Mekhan.,50,1,155-159,1986